8. M. V. Stepanenko, "On the dynamics of unidirectional composite rupture," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1979).

ASYMPTOTIC ANALYSIS OF LONGITUDINAL AND BENDING WAVES PROPAGATED IN A SYSTEM OF TWO PLATES FASTENED AT AN ANGLE

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1. We introduce two coordinate systems so that in the left plate $x_1 \le 0$ while in the right $x_2 \ge 0$ (Fig. 1). The z_1 and z_2 axes are here directed normally to the plate surfaces so that the two half-planes ($z_1 = 0$ for $x_1 \le 0$ and $z_2 = 0$ for $x_2 \ge 0$) would coincide with their neutral planes. The y axis is along the line connecting the plates. Let an incident sinusoidal wave be propagated in the left plate. We examine the conditions for its passage through the boundary.

The plate vibrations are described by the following differential equations

$$D(\partial^4 w/\partial x^4 + 2\partial^4 w/\partial x^2\partial y^2 + \partial^4 w/\partial y^4) + \rho h \partial^2 w/\partial t^2 = 0; \tag{1.1}$$

$$\frac{Eh}{1-\nu^2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x \partial y}\right) = \rho h \frac{\partial^2 u}{\partial t^2}; \tag{1.2}$$

$$\frac{Eh}{1-\nu^2}\left(\frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x \partial y}\right) = \rho h \frac{\partial^2 v}{\partial t^2}, \tag{1.3}$$

where h is the thickness of the plates; E, elastic modulus; ν, Poisson ratio; and D, bending stiffness.

Displacements v_1, v_2 of points of the plate neutral planes along the y axis during vibrations u_1, u_2 along the x_1, x_2 axes will characterize the wave in the planes of the plates while the displacements w_1, w_2 along the z_1, x_2 axes, respectively, are bending waves [1, 2].

Solutions of the problem should satisfy eight boundary conditions on the hinge-supported edges

$$w = u = \partial v/\partial y = \partial^2 w/\partial y^2 = 0 \quad (y = 0,\ l) \tag{1.4}$$

and eight juncture conditions on a common edge (x = 0)

$$u_1 = u_2 \cos\varphi + w_2 \sin\varphi; \tag{1.5}$$

$$w_1 = -u_2 \sin\varphi + w_2 \cos\varphi; \tag{1.6}$$

$$\partial w_1/\partial x_1 = \partial w_2/\partial x_2; \tag{1.7}$$

$$\sigma_{x_1} = \sigma_{x_2}\cos\varphi + R_{x_2}\sin\varphi; \tag{1.8}$$

$$R_{x_1} = -\sigma_{x_2}\sin\varphi + R_{x_2}\cos\varphi; \tag{1.9}$$

$$M_{x_1} = M_{x_2}; \tag{1.10}$$

$$v_1 = v_2; \tag{1.11}$$

$$\tau_{x_1 y} = \tau_{x_2 y}, \tag{1.12}$$

where $\varphi = \pi - \alpha$ (α is the angle between the plates), σ_{x_i} are the normal stresses, $\tau_{x_i y}$ are the shear stresses, M_{x_i} are bending moments relative to the x = 0 axis, $Q_{x_i} + \partial M_{x_i y}/\partial y$ (Q_{x_i} is the transverse force, $M_{x_i y}$ is the torque), i = 1, 2.

2. Let us first consider the particular case of the plane problem ($l = \infty$). Then v = 0, the solutions are independent of the variable y and the boundary conditions (1.4) drop out.

The juncture conditions on the common edge have the following form in this case

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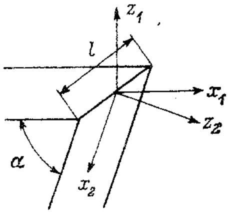

Fig. 1

$$u_1 = u_2\cos\varphi + w_2\sin\varphi,\ w_1 = -u_2\sin\varphi + w_2\cos\varphi,\ \partial w_1/\partial x_1 = \partial w_2/\partial x_2,$$

$$\frac{E_1h_1}{1-\nu_1^2}\frac{\partial u_1}{\partial x_1} = \frac{E_2h_2}{1-\nu_2^2}\frac{\partial u_2}{\partial x_2}\cos\varphi - \frac{E_2h_2^3}{12\left(1-\nu_2^2\right)}\frac{\partial^3 w_2}{\partial x_2^3}\sin\varphi,$$

$$\frac{E_1h_1^3}{12\left(1-\nu_1^2\right)}\frac{\partial^3 w_1}{\partial x_1^3} = \frac{E_2h_2}{1-\nu_2^2}\frac{\partial u_2}{\partial x_2}\sin\varphi + \frac{E_2h_2^3}{12\left(1-\nu_2^2\right)}\frac{\partial^3 w_2}{\partial x_2^3}\cos\varphi,$$

$$\frac{E_1h_1^3}{1-\nu_1^2}\frac{\partial^2 w_1}{\partial x_1^2} = \frac{E_2h_2^3}{1-\nu_2^2}\frac{\partial^2 w_2}{\partial x_2^2}.$$

Let us reduce (1.1)-(1.3) to dimensionless form. To do this we introduce the dimensionless coordinates $\xi = x/h_1$, $\eta = y/h_1$, the dimensionless variables $u^* = u/h_1$, $v^* = v/h_1$, $w^* = w/h_1$, and the dimensionless time $\tau = ct/h_1 = \sqrt{E/\rho(1-\nu^2)}(t/h_1)$.

We then obtain (for each of the plates)

$$\frac{\partial^2 u^*}{\partial \xi^2} + \frac{1-\nu}{2}\frac{\partial^2 u^*}{\partial \eta^2} + \frac{1+\nu}{2}\frac{\partial^2 v^*}{\partial \xi \partial \eta} = \frac{\partial^2 u^*}{\partial \tau^2}; \tag{2.1}$$

$$\frac{1-\nu}{2}\frac{\partial^2 v^*}{\partial \xi^2} + \frac{\partial^2 v^*}{\partial \eta^2} + \frac{1+\nu}{2}\frac{\partial^2 u^*}{\partial \xi \partial \eta} = \frac{\partial^2 v^*}{\partial \tau^2}; \tag{2.2}$$

$$\Delta\Delta w^* + 12(h_1/h_k)^2 \partial^2 w^*/\partial \tau^2 = 0. \tag{2.3}$$

Here k=1 or 2 depending onto which plate we apply the equation $\Delta w^* = \partial^2 w^*/\partial \xi^2 + \partial^2 w^*/\partial \eta^2$.

We write the plate juncture conditions in the case $E_1 = E_2$, $\nu_1 = \nu_2$ as

$$u_1^* = u_2^*\cos\varphi + w_2^*\sin\varphi; \tag{2.4}$$

$$w_1^* = -u_2^*\sin\varphi + w_2^*\cos\varphi; \tag{2.5}$$

$$\partial w_1^*/\partial \xi_1 = \partial w_2^*/\partial \xi_2; \tag{2.6}$$

$$\frac{\partial u_1^*}{\partial \xi_1} = \frac{h_2}{h_1}\frac{\partial u_2^*}{\partial \xi_2}\cos\varphi - \frac{1}{12}\left(\frac{h_2}{h_1}\right)^3\frac{\partial^3 w_2^*}{\partial \xi_2^3}\sin\varphi; \tag{2.7}$$

$$\frac{1}{12}\frac{\partial^3 w_1^*}{\partial \xi_1^3} = \frac{h_2}{h_1}\frac{\partial u_2^*}{\partial \xi_2}\sin\varphi + \frac{1}{12}\left(\frac{h_2}{h_1}\right)^3\frac{\partial^3 w_2^*}{\partial \xi_2^3}\cos\varphi; \tag{2.8}$$

$$\frac{\partial^2 w_1^*}{\partial \xi_1^2} = \left(\frac{h_2}{h_1}\right)^3\frac{\partial^2 w_2^*}{\partial \xi_2^2}. \tag{2.9}$$

Let the incident waves have the form

$$w^* = w_{01}^* e^{i\left(\omega^*\tau - \nu_1^*\xi_1\right)}; \tag{2.10}$$

$$u^* = u_{01}^* e^{i\left(\omega^*\tau - \alpha_1^*\xi_1\right)}, \tag{2.11}$$

where $\omega^* = \left(\frac{\omega h_1}{c}\right)\frac{1}{\sqrt{12}}$; , w_{01}^*, u_{01}^* are dimensionless input amplitudes

$$w_{01}^* = w_{01}/h_1; \quad u_{01}^* = u_{01}/h_1$$

(we consider the amplitudes w_{01}, u_{01} given).

Part of the wave being propagated over the left plate will pass through the edge to the right, and part of the wave will be reflected backward.

For the left plate the reflected bending wave has the form

$$w^* = w_1^* e^{i\left(\omega^*\tau + \gamma_1^*\xi_1\right)} + w_1^* e^{\gamma_1^*\xi_1} e^{i\omega^*\tau}, \tag{2.12}$$

where the first term is a harmonic wave and the second is an edge effect.

The longitudinal reflected wave has the form

$$u = u_1^* e^{i\left(\omega^*\tau + \alpha_1^*\xi_1\right)}. \tag{2.13}$$

For the right plate we have

$$w^* = w_2^* e^{i\left(\omega^*\tau - \gamma_2^*\xi_2\right)} + w_2'^* e^{-\gamma_2^*\xi_2} e^{i\omega^*\tau}, \quad u^* = u_2^* e^{i\left(\omega^*\tau - \alpha_2^*\xi_2\right)}. \tag{2.14}$$

The values of γ_k^*, α_k^* are found from (2.1) and (2.3) upon substitution of solutions in the above-mentioned form:

$$\gamma_k^* = \sqrt[4]{12}\sqrt{\frac{h_1}{h_k}\omega^*}, \quad \alpha_k^* = \omega_k^* \quad (k = 1, 2).$$

Substitution of (2.10)-(2.14) in (2.4)-(2.9) yields the following system of six linear equations (with six unknown dimensionless amplitudes w_1^*, $w_1'^*$, w_2^*, $w_2'^*$, u_1^*, u_2^*):

$$u_{01}^* + u_1^* = u_2^* \cos\varphi + (w_2^* + w_2'^*)\sin\varphi; \tag{2.15}$$

$$w_{01}^* + w_1^* + w_1'^* = -u_2^* \sin\varphi + (w_2^* + w_2'^*)\cos\varphi; \tag{2.16}$$

$$\gamma_1^*(-iw_{01}^* + iw_1^* + w_1'^*) = \gamma_2^*(-iw_2^* - w_2'^*); \tag{2.17}$$

$$\alpha_1^*(-iu_{01}^* + iu_1^*) = -\alpha_2^*\left(\frac{h_2}{h_1}\right) iu_2^*\cos\varphi + \frac{1}{12}(\gamma_2^*)^3\left(\frac{h_2}{h_1}\right)^3(-iw_2^* + w_2'^*)\sin\varphi; \tag{2.18}$$

$$\frac{1}{12}(\gamma_1^*)^3(-iw_{01}^* + iw_1^* - w_1'^*) = \alpha_2^*\left(\frac{h_2}{h_1}\right) iu_2^*\sin\varphi + \frac{1}{12}(\gamma_2^*)^3\left(\frac{h_2}{h_1}\right)^3(-iw_2^* + w_2'^*)\cos\varphi; \tag{2.19}$$

$$(\gamma_1^*)^2(-w_{01}^* - w_1^* + w_1'^*) = (\gamma_2^*)^2\left(\frac{h_2}{h_1}\right)^3(-w_2^* + w_2'^*). \tag{2.20}$$

We shall seek the asymptotic solution of the system (2.15)-(2.20) under the assumption that the angle α between the plates is not small nor close to π.

Let us introduce the parameter $k = \sqrt{\omega^*} = \frac{1}{\sqrt[4]{12}}\sqrt{\frac{\omega h_1}{c}}$, which is henceforth considered small. Let us use the notation $h_1/c = T_0$, $2\pi/\omega = T_\omega$. The smallness of the parameter k means that the time for the wave to pass the distance h is much less than the period $2\pi/\omega$.

After dividing both sides of (2.19) by the quantity $\alpha_2^* h_2/h_1$, we obtain

$$\frac{1}{\sqrt[4]{12}}\frac{h_1}{h_2}\omega^{*1/2}\left[-iw_{01}^* + iw_1^* - w_1'^*\right] = iu_2^*\sin\varphi + \frac{1}{\sqrt[4]{12}}\left(\frac{h_2}{h_1}\right)^{1/2}\omega^{*1/2}(-iw_2^* + w_2'^*),$$

from which there follows in the zero approximation ($\sqrt{\omega^*} = k = 0$)

$$u_2^* = 0 \quad (\sin\varphi \neq 0). \tag{2.21}$$

Dividing (2.18) by α_1^* and taking (2.21) into account, we obtain

$$-iu_{01}^* + iu_1^* = \frac{1}{\sqrt[4]{12}}\left(\frac{h_2}{h_1}\right)^{3/2}\omega^{*1/2}(-iw_2^* + w_2'^*)\sin\varphi,$$

from which

$$u_1^* = u_{01}^*. \tag{2.22}$$

Taking account of (2.21) and (2.22), equations (2.15) and (2.16) are converted as follows

$$w_2'^* = -w_2^* + \frac{2}{\sin\varphi}u_{01}^*; \tag{2.23}$$

$$w_1'^* = -w_{01}^* - w_1^* + 2\frac{\cos\varphi}{\sin\varphi}u_{01}^* \tag{2.24}$$

Substituting (2.23) and (2.24) into (2.17) and (2.20), and transposing the terms containing the known dimensionless amplitudes u_{01}^*, w_{01}^* to the right side, we obtain a system of two linear equations with two unknowns w_1^*, w_2^*, whose solution has the form

$$w_1^* = -\frac{\left[1+i\left(\frac{h_2}{h_1}\right)^{5/2}\right]}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} w_{01}^* + \frac{\left\{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]\cos\varphi + i\left(\frac{h_2}{h_1}\right)^{5/2}\left[\cos\varphi + \left(\frac{h_1}{h_2}\right)^{1/2}\right]\right\}\frac{1}{\sin\varphi}}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} u_{01}^*,$$

$$w_2^* = -\frac{(i-1)\left(\frac{h_2}{h_1}\right)^{1/2}}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} w_{01}^* + \frac{\left\{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right] + i\left[\left(\frac{h_2}{h_1}\right)^{1/2}\cos\varphi + 1\right]\right\}\frac{1}{\sin\varphi}}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} u_{01}^*.$$

Therefore, in a zero approximation the longitudinal wave is reflected entirely from the rib as from a free edge and is returned backward; it here generates two bending waves. The bending perturbation generates just bending waves, where the transfer of the perturbation from the bending wave is independent of the angle between the plates (under the assumption that $\varphi \neq 0$, $\varphi \neq \pi$). The solution is independent of the vibration frequency ω. For $\varphi = \pi/2$ we have

$$w_1^* = -\frac{\left[1+i\left(\frac{h_2}{h_1}\right)^{5/2}\right]}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} w_{01}^* + \frac{i\left(\frac{h_2}{h_1}\right)^2}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} u_{01}^*,$$

$$w_2^* = -\frac{(i-1)\left(\frac{h_2}{h_1}\right)^{1/2}}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} w_{01}^* + \left\{1 + \frac{i}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]}\right\} u_{01}^*, \quad u_1^* = u_{01}^*, \quad u_2^* = 0.$$

3. Let us examine the question of the transfer of wave energy. We write the formulas describing the wave moving in the x direction

$$w = w_0 \sin(\omega t + \gamma x), \quad u = u_0 \sin(\omega t + \alpha x). \tag{3.1}$$

We take the bending wave and we compute the kinetic and potential energy on some of its length

$$0 \longleftarrow -\frac{2\pi}{\gamma},$$

$$T_w = \frac{\rho h_k}{2}\int_s (w_t')^2\, ds = \frac{\rho h_k h_1 c^2}{2}\int_{s^*} (w_\tau'^*)\, ds^* = \frac{\rho h_k h_1 c^2}{2} |w_0^*|^2 (w^*)^2 \frac{\pi}{\gamma_k^*},$$

$$\Pi_w = \frac{D}{2}\int_s \left(\frac{\partial^2 w}{\partial x^2}\right)^2 ds = \frac{\rho h_k^3 c^2}{2\cdot 12 h_1}\int_{s^*}\left(\frac{\partial^2 w^*}{\partial \xi^2}\right)^2 ds^* = \frac{\rho h_k^3 c^2}{2\cdot 12 h_1} |w_0^*|^2 (\gamma_k^*)^4 \frac{\pi}{\gamma_k^*},$$

$$k = 1, 2, \quad s^* = s/h_1.$$

Substituting (3.1) (in dimensionless form) into (2.3) yields the relationship $(\gamma^*)^4 = 12(h_1/hk)^2(\omega^*)^2$ which means that the kinetic energy of the plate bending wave equals its potential energy $T_W = \Pi_w$.

It can be shown analogously that $T_u = \Pi_u$.

Now, in order to verify satisfaction of the energy balance $E_0 = E_1 + E_2$, it is sufficient to show that $T_0 = T_1 + T_2$.

For simplicity, we consider that only the bending perturbation $w^* = w_{01}^* e^{i(w^*\tau - \gamma_1^*\xi)}$ is delivered to the left plate.

In the zeroth approximation this wave generates only bending waves with the dimensionless amplitudes w_1^*, w_2^*, which are determined from the formulas

$$w_1^* = -\frac{\left[1+i\left(\frac{h_2}{h_1}\right)^{5/2}\right]}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} w_{01}^*, \quad w_2^* = -\frac{(i-1)\left(\frac{h_2}{h_1}\right)^{1/2}}{\left[1+\left(\frac{h_2}{h_1}\right)^{5/2}\right]} w_{01}^*.$$

Let us examine the reduced (dimensionless) kinetic energy of the plate bending wave. To do this, we divide T_W by the quantity $\rho c^2 h_1^2$

$$T_w^* = \frac{1}{2}\frac{h_k}{h_1}|w_0^*|^2 (\omega^*)^2 \frac{\pi}{\gamma_k^*}, \quad k=1, 2.$$

The energy $T^*_{w_0} = \frac{1}{2} |w^*_{01}|^2 (\omega^*)^2 \frac{\pi}{\gamma^*_1}$, was fed to the edge along the left plate, while the energy

$$T^*_w = T^*_{w_1} + T^*_{w_2} = \frac{1}{2} |w^*_1|^2 (\omega^*)^2 \frac{\pi}{\gamma^*_1} + \frac{1}{2} |w^*_2|^2 (\omega^*)^2 \frac{\pi}{\gamma^*_2} \frac{h_2}{h_1},$$

$$\gamma^*_2 = \gamma^*_1 \left(\frac{h_1}{h_2}\right)^{1/2},$$

$$|w^*_1|^2 + |w^*_2|^2 \left(\frac{h_2}{h_1}\right)^{3/2} = \frac{1 + \left[\left(\frac{h_2}{h_1}\right)^{5/2}\right]^2 + 2\frac{h_2}{h_1}\left(\frac{h_2}{h_1}\right)^{3/2}}{\left[1 + \left(\frac{h_2}{h_1}\right)^{5/2}\right]^2} |w^*_{01}|^2 = |w^*_{01}|^2.$$

Therefore, the energy balance is satisfied.

Let us designate the quantity $\left(\frac{h_2}{h_1}\right)^{3/2} |w^*_2|^2$ the amplitude coefficient of bending wave energy passage, and the quantity $|w^*_1|^2$ as amplitude reflection coefficient:

$$K_{\text{pa}} = \left(\frac{h_2}{h_1}\right)^{3/2} |w^*_2|^2, \quad K_{\text{ref}} = |w^*_1|^2, \quad K_{\text{pa}} + K_{\text{ref}} = 1$$

(such an input perturbation w_{01} is taken that $|w^*_{01}|^2 = 1$).

In order for the major part of the energy to be reflected from the edge, the condition $K_{\text{paw}} > K_{\text{paw}}$ or $\left[1 - \left(\frac{h_2}{h_1}\right)^{5/2}\right]^2 > 0$ must be satisfied.

Therefore, in a zeroth approximation (k=0), when the plates are of identical thickness, half the energy passes through the edge; if the plate thicknesses are distinct, then the major part of the energy is always returned (the assumption that the angle between the plates is not small and close to π is retained).

Limit cases: 1. The right plate is much thinner than the left $h_2/h_1 \to 0$, then $K_{\text{ref}\,w} = |w_1|^2 \sim 1$, $K_{\text{pa}w} \sim (h_2/h_1)^{5/2} \to 0$, i.e., in practice all the energy is returned backward (Fig. 2).

2. Left plate is much thinner than the right

$$h_1/h_2 \to 0, \quad K_{\text{ref}w} \sim 1, \quad K_{\text{pa}w} \sim (h_1/h_2)^{5/2} \to 0.$$

In this case almost all the energy is also returned backward (Fig. 3).

If the angle between the plates is $\alpha = \pi\,(\varphi = 0)$, then the zeroth approximation yields the following expressions for the amplitudes

$$u^*_1 = u^*_2 = 0,$$

$$w^*_1 = \frac{4\left(\frac{h_2}{h_1}\right)\left(\frac{h_2}{h_1} - 1\right) - 2\left(\frac{h_2}{h_1}\right)^{1/2}\left[\left(\frac{h_2}{h_1}\right)^{3/2} - \left(\frac{h_1}{h_2}\right)^{1/2}\right]^2 i}{\left\{\left[\left(\frac{h_2}{h_1}\right)^{3/2} + \left(\frac{h_1}{h_2}\right)^{1/2}\right]\left[1 + \left(\frac{h_2}{h_1}\right)^{1/2}\right]\right\}^2 - \left[\left(\frac{h_2}{h_1}\right)^{3/2} - \left(\frac{h_1}{h_2}\right)^{1/2}\right]^2 \quad \left(1 + \frac{h_2}{h_1}\right)} w^*_{01},$$

$$w^*_2 = \frac{4\left[\left(\frac{h_2}{h_1}\right)^{3/2} + \left(\frac{h_1}{h_2}\right)^{1/2}\right]\left[1 + \left(\frac{h_2}{h_1}\right)^{1/2}\right]}{\left\{\left[\left(\frac{h_2}{h_1}\right)^{3/2} + \left(\frac{h_1}{h_2}\right)^{1/2}\right]\left[1 + \left(\frac{h_2}{h_1}\right)^{1/2}\right]\right\}^2 - \left[\left(\frac{h_2}{h_1}\right)^{3/2} - \left(\frac{h_1}{h_2}\right)^{1/2}\right]^2} - \frac{}{\left(1 + \frac{h_2}{h_1}\right)} w^*_{01}.$$

Here $K_{\text{refw}} \to 0$, $K_{\text{paw}} \sim 1$ for plates of identical thickness, while if $h_2/h_1 \to 0$ or $h_1/h_2 \to 0$, $K_{\text{refw}} \sim 1$, $K_{\text{paw}} \to 0$.

4. Let us turn to the general case when vibrations occur in all three directions. The wave in the x axis direction (in dimensionless form) is described by the formulas

$$u^*(\xi, \eta) = u^* \sin \lambda^*\eta \sin (\alpha^*\xi - \omega^*\tau); \tag{4.1}$$

$$v^*(\xi, \eta) = v^* \cos \lambda^*\eta \cos (\alpha^*\xi - \omega^*\tau); \tag{4.2}$$

$$w^*(\xi, \eta) = w^* \sin \lambda^*\eta \sin (\gamma^*\xi - \omega^*\tau), \tag{4.3}$$

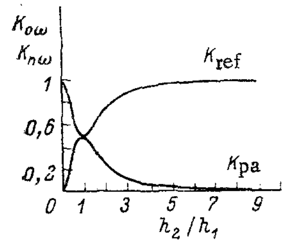

Fig. 2

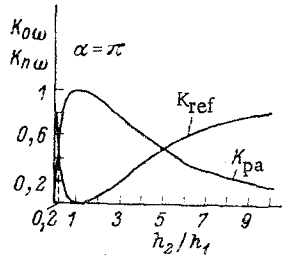

Fig. 3

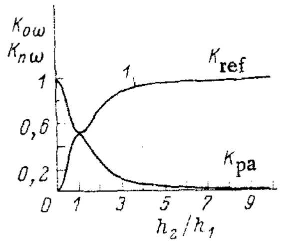

Fig. 4

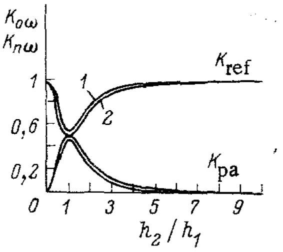

Fig. 5

where $\lambda^* = \lambda h_1$, and the parameter λ is such that an integer number of half-waves $l(\pi/\lambda) = n$ is packed along a common edge of length l, where n is an integer.

Substituting (4.3) into (2.3), we obtain

$$(\gamma_k^{*2} + \lambda^{*2})^2 = 12(h_1/h_k)^2\omega^{*2},\ k = 1, 2,\ \text{where}\ (\gamma^*)^2_{1,2} = \pm\frac{h_1}{h_2}\sqrt{12}\,\omega^* - \lambda^{*2}.$$

Substituting (4.1) and (4.2) into (2.1) and (2.2), we obtain a homogeneous system of two linear equations with two unknowns u^* and v^*:

$$\left(\alpha^{*2} + \frac{1-\nu}{2}\lambda^{*2} - \omega^{*2}\right)u^* - \frac{1+\nu}{2}\alpha^*\lambda^*v^* = 0,\ -\frac{1+\nu}{2}\alpha^*\lambda^*u^* + \left(\frac{1-\nu}{2}\alpha^{*2} + \lambda^{*2} - \omega^{*2}\right)v^* = 0. \tag{4.4}$$

The equality of the determinant of this system to zero yields a quadratic equation in ω^{*2}:

$$(\omega^{*2})^2 - [(3-\nu)/2](\alpha^{*2} + \lambda^{*2})\omega^{*2} + [(1-\nu)/2](\alpha^{*2} + \lambda^{*2})^2 = 0. \tag{4.5}$$

Equation (4.5) has the roots

$$(\omega^{*2})_1 = \alpha^{*2} + \lambda^{*2},\ (\omega^{*2})_2 = [(1-\nu)/2](\alpha^{*2} + \lambda^{*2}), \tag{4.6}$$

where the solution of the system (4.4) $u^* = \alpha^*$, $v^* = -\lambda^*$ corresponds to the first root, and $u^* = \lambda^*$, $\omega = \alpha^*$. to the second. We have from (4.6)

$$(\alpha^{*2})_1 = \omega^{*2} - \lambda^{*2},\ (\alpha^{*2})_2 = [2/(1-\nu)]\omega^{*2} - \lambda^{*2}.$$

Let us introduce the notation

$$(\gamma^*)^2 = (\gamma^{*2})_1 = (h_1/h_k)\sqrt{12}\,\omega^* - \lambda^{*2},\ (\gamma'^*)^2 = -(\gamma^{*2})_2 = (h_1/h_2)\sqrt{12}\,\omega^* + \lambda^{*2},$$
$$(\alpha^*)^2 = -(\alpha^{*2})_1 = \lambda^{*2} - \omega^{*2},\ (\alpha'^*)^2 = -(\alpha^{*2})_2 = \lambda^{*2} - [2/(1-\nu)]\omega^{*2},$$

if

$$(h_1/h_k)\sqrt{12}\,\omega^* > \lambda^{*2},\ \omega^* > \lambda^*, \tag{4.7}$$

then γ^*, γ'^* are real and α^*, α'^* are pure imaginary. We consider that just the bending perturbation $w^* = \sin\lambda^*\eta w^*_{01} e^{i(\omega^*\tau - \gamma_1^*\xi_1)}$ with the given amplitude w_{01} ($w^*_{01} = w_{01}/h_1$) is delivered to the input. We write the solution at the output in the following form:

Left plate

$$w^* = \sin\lambda^*\eta\left[w_1^* e^{i(\omega^*\tau + \gamma_1^*\xi_1)} + w_1'^* e^{i\omega^*\tau + \gamma_1'^*\xi_1}\right],$$
$$u^* = \sin\lambda^*\eta\left[u_1^*\alpha_1^* e^{i\omega^*\tau} e^{\alpha_1^*\xi_1} + u_1'^*\lambda^* e^{i\omega^*\tau} e^{\alpha_1'^*\xi_1}\right],$$
$$v^* = \cos\lambda^*\eta\left[u_1^*\lambda^* e^{i\omega^*\tau} e^{\alpha_1^*\xi_1} + u_1'^*\alpha_1'^* e^{i\omega^*\tau} e^{\alpha_1'^*\xi_1}\right];$$

Right plate

$$w^* = \sin\lambda^*\eta\left[w_2^* e^{i(\omega^*\tau+\gamma_2^*\xi_2)} + w_2'^* e^{i\omega^*\tau-\gamma_2'^*\xi_2}\right],$$

$$u^* = \sin\lambda^*\eta\left[-u_2^*\alpha_2^* e^{i\omega^*\tau} e^{-\alpha_2^*\xi_2} + u_2'^*\lambda^* e^{i\omega^*\tau} e^{-\alpha_2'^*\xi_2}\right],$$

$$v^* = \cos\lambda^*\eta\left[u_2^*\lambda^* e^{i\omega^*\tau} e^{-\alpha_2^*\xi_2} - u_2'^*\alpha_2'^* e^{i\omega^*\tau} e^{-\alpha_2'^*\xi_2}\right],$$

hence, the wave occurs in the same domain where condition (4.7) is satisfied. The solution in the above-mentioned form satisfies the eight boundary conditions (1.4). Substituting this solution in the juncture conditions on a common edge (1.5)-(1.12) (in the zeroth approximation) under the assumption that $\varphi \sim 0\,(k^0)$, $(\pi-\varphi) \sim 0)k^0)$, we obtain that the bending perturbation generates only bending waves with the amplitudes

$$w_1^* = -\frac{\left\{\left[(\gamma_1^*)^2+(\gamma_1'^*)^2\right]\gamma_2'^* + \left(\frac{h_2}{h_1}\right)^3\left[(\gamma_2^*)^2+(\gamma_2'^*)^2\right]\gamma_1'^*\right\} + i\left\{-\left[(\gamma_1^*)^2+(\gamma_1'^*)^2\right]\gamma_2^* + \left(\frac{h_2}{h_1}\right)^3\left[(\gamma_2^*)^2+(\gamma_2'^*)^2\right]\gamma_1^*\right\}}{\left\{\left[(\gamma_1^*)^2+(\gamma_1'^*)^2\right]\gamma_2'^* + \left(\frac{h_2}{h_1}\right)^3\left[(\gamma_2^*)^2+(\gamma_2'^*)^2\right]\gamma_1'^*\right\} + i\left\{\left[(\gamma_1^*)^2+(\gamma_1'^*)^2\right]\gamma_2^* + \left(\frac{h_2}{h_1}\right)^3\left[(\gamma_2^*)^2+(\gamma_2'^*)^2\right]\gamma_1^*\right\}}\,w_{01}^*, \quad (4.8)$$

$$w_2^* = -\frac{2i\left[(\gamma_1^*)^2+(\gamma_1'^*)^2\right]\gamma_1^*}{\left\{\left[(\gamma_1^*)^2+(\gamma_1'^*)^2\right]\gamma_2'^* + \left(\frac{h_2}{h_1}\right)^3\left[(\gamma_2^*)^2+(\gamma_2'^*)^2\right]\gamma_1'^*\right\} + i\left\{\left[(\gamma_1^*)^2+(\gamma_1'^*)^2\right]\gamma_2^* + \left(\frac{h_2}{h_1}\right)^3\left[(\gamma_2^*)^2+(\gamma_2'^*)^2\right]\gamma_1^*\right\}}\,w_{01}^*,$$

which are independent (as in the plane problem) of the angle between the plates.

If we set $\gamma_i^* = \gamma_i'^* = (i=1,\,2)\,(\lambda=0)$, which corresponds to the case when there are no vibrations in the y direction (infinitely wide plates), then (4.8) is reduced to the same results as in Sec. 2.

Executing calculations analogous to the calculations in Sec. 3, we obtain that even in this case the kinetic energy of the plate bending wave equals its potential energy:

$$T_w^* = \Pi_w^* = \frac{1}{2}\frac{h_k}{h_1}\,|w_0^*|^2(\omega^*)^2\frac{\pi}{\gamma_k^*}\frac{\pi}{\lambda^*},\ k=1,2,\ K_{\mathrm{pa}_w} = |w_1^*|^2,$$

$$K_{\mathrm{ref}_w} = \left(\frac{h_2}{h_1}\right)^3\frac{\gamma_2^*}{\gamma_1^*}\frac{(\gamma_2^*)^2+(\gamma_2'^*)^2}{(\gamma_1^*)^2+(\gamma_1'^*)^2}\,|w_2^*|^2.$$

All the exposition above is true for $\sqrt{\omega^*} \ll 1$.

Graphs of the dependences of the bending wave reflection and passage coefficients on the ratio between the plate thicknesses are given in Figs. 4 and 5 for different values of the parameters ω^* and λ^* (Fig. 4: $\omega^* = 0.01$, line 1) $\lambda^* = 0.005$; 0.009; (Fig. 5: $\omega^* = 0.1$; line 1) $\lambda^* = 0.09$, 2) $\lambda^* = 0.005$; 0.009; 0.02). It is seen from Figs. 4 and 5 that the curves differ negligibly from the curves in Fig. 2.

Therefore, if the parameter ω^* is sufficiently small, the plane problem, which is simpler, can be considered instead of the general case.

LITERATURE CITED

1. S. P. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill (1959).
2. Wave Propagation in Structures from Thin Rods and Plates [in Russian], Nauka, Moscow (1974).